

Voting power in the European Union

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1 Introduction

Weighted voting games are mathematical models that are used to analyze the distribution of the decision power of each country within an international organization such as the United Nations or the European Union. In these institutions each nation has a number of votes and a proposal is approved if it has the support of a coalition of countries with a number of votes higher than a preestablished limit. For instance, the voting method in the United Nations Security Council, formed by five standing members (USA, Russia, China, UK and France) and ten temporary members. Each one of the standing members has seven votes and the temporary ones have only one vote each one. There are necessary at least 39 votes to approve a proposal. Let us notice that each coalition without one of the permanent members would get at most $(4 \times 7) + 10$ votes, a lower number than the fixed amount of votes needed to pass the proposal. Then, we say that the permanent members have the authority to veto any proposal, so we say that they are veto players in this voting game.

Recently we have seen a big controversy when deciding a voting system within the European Union. In this paper we try to give answers about why some countries want to modify the Nice treaty while others do not.

2 Voting systems in the EU

In the Treaty of Nice a voting system consisting of giving to each country a number of votes and fixing a qualified majority in order to approve a proposal was given.

In table 1 we have the allocations of votes decided after the Treaty of Nice. After the complains of several countries, that voting system was not approved, and it became the biggest problem to create the first European Constitution.

It was proposed another system, which said that a proposal needed the support of, at least, 13 countries out of 25 with, at least, some 60% of the total population to be approved. This system of double majority also gave problems, since other countries preferred the Treaty of Nice.

The third voting system proposed up till now, and the one that was approved in the Summit in Brussels last 18th July 2004 to be included in the European Constitution, is also based in a double majority system. It is necessary to have 15 countries with at least some 65% of the population in order to get a proposal approved. In addition, the necessary number of nations to block a bill in the parliament is fixed in four, and the abstentions do not count at all.

State	Weighted votes
Germany	29
United Kingdom	29
France	29
Italy	29
Spain	27
Poland	27
The Netherlands	13
Greece	12
Czech Republic	12
Belgium	12
Hungary	12
Portugal	12
Sweden	10
Austria	10
Slovakia	7
Denmark	7
Finland	7
Ireland	7
Lithuania	7
Latvia	4
Slovenia	4
Estonia	4
Cyprus	4
Luxembourg	4
Malta	3
Total	321
Qualified majority	232

Table 1: Allocation of votes in the Treaty of Nice.

Once we have described those three voting systems, we will try to find an explanation for some countries wanted to change those systems and others did not.

3 Banzhaf power index

Let us suppose that we have a set of countries within an international organization, such as the UN or the EU. Let us denote that set as N . Given a fixed country $i \in N$, we define a *swing for a country i* as the pair of sets of the form

$$(S, S - \{i\})$$

where S is a group of countries that contains i and is a winning coalition, and $S - \{i\}$ is not a winning coalition.

We say that a coalition of countries is a *winning coalition* if it has the power of approving any proposal by itself, without taking into account what the remaining countries vote. For each country i we denote by δ_i the number of swings for the country i . We shall write $\bar{\delta}$ for the total number of swings, i.e., $\bar{\delta} = \sum_{i \in N} \delta_i$. A country with $\delta_i = 0$ is called a *dummy* because, intuitively, he can never help a coalition to win. On the other hand, a country with $\delta_i = \bar{\delta}$ is called a *dictator*, for he is the only one that can make a coalition be a winning coalition.

The swing numbers δ_i will be called the “raw” Banzhaf indices. These are the indices that Banzhaf actually defined and used in his work. But since the principal interest in these numbers lies in their ratios rather than their magnitudes, it has been common practice to normalize them to add up to 1:

$$\beta_i = \frac{\delta_i}{\bar{\delta}}, \forall i \in N.$$

In the following example we see how the Banzhaf index works:

Example 3.1 *There is a international organization, called ORG, made up of four countries, called A, B, C and D. Let us suppose that in order to approve a law in the council of ORG they are necessary at least 51 votes. The table of allocation votes for each country is:*

Country	A	B	C	D
Number of votes	40	30	20	10

Let us calculate the Banzhaf indices for each country. To do that, we have to calculate the number of swings for each country i , that is to say, the number of coalitions of countries that are winners with i and are not winners without i .

- If the country A joins to the country B , they would form a winning coalition, (A, B) , and B was not a winning coalition by itself. So, B is a swing for A . It happens the same if A joins to C , (B, C) , (B, D) or (C, D) . That is, the country A has 5 swings, $\delta_A = 5$.
- The non winning coalitions that B can make winners are: A , (A, D) and (C, D) . Note that (A, C) is not a swing for B , since (A, C) is already a winning coalition. So B has 3 swings, $\delta_B = 3$.
- The swings of C are A , (A, D) and (B, D) . So C has 3 swings, $\delta_C = 3$.
- D only has one swing, (B, C) , $\delta_D = 1$.

To summarize, we have that the vector of “raw” Banzhaf indices is:

$$(\delta_A, \delta_B, \delta_C, \delta_D) = (5, 3, 3, 1),$$

and dividing it by $\bar{\delta} = 12$ we get

$$(\beta_A, \beta_B, \beta_C, \beta_D) = (0.416, 0.25, 0.25, 0.083).$$

The number of swings in which each country participates allows us to assign them a power index, that measures their capacity to reach coalitions that overcome a fixed share to approve decisions. That is, the power of a nation is measured by calculating the times that their votes change a coalition that did not reach the share into a winning coalition. The Banzhaf index gives us a more accurate measure of the power that a country has than the number of votes that it has, because those two concepts not always go together, as shown in the following example.

Example 3.2 Let us suppose a union of nations that have the following allocation of votes

A	B	D
49	49	2

having the rule that a proposal is approved if it has the support of, at least, 51 votes. It seems that the countries A and B have got more power than C , but if one calculates their Banzhaf indices one gets that all of them are equal to $\frac{1}{3}$. That means that all of them have the same power !!!

For reasons like the one we saw in last example some countries are not happy with some voting systems. We will find out the power indices of the European countries in the European Council and see why some nations wanted to change the voting system and others did not. But before that let us propose two exercises to practice with the Banzhaf indices.

Exercise 3.1 *Calculate the Banzhaf indices with the data of example 3.1 changing the voting systems to:*

1. *To approve a law it is necessary the support of more than some 65% of the votes.*
2. *To approve a law it is necessary the support of at least three countries with more than some 65% of the votes.*

Which is the best voting system for each country? Why?

Exercise 3.2 *Find the vector of power indices for the voting system of the Security Council of the United Nations, system shown in the introduction of the paper.*

4 Power indices in the EU council

In this section we find out the Banzhaf power indices for the EU countries in the European Council for the three voting systems we saw in section 2. To do that we first need to know the data of the population¹ of every EU country, what is shown in table 2.

The calculations of the Banzhaf power indices for those 25 countries are huge, and so we are going to left them out, but the way to do them is as we showed in previous sections. Since they are so tedious, impossible to make it by hand, the way to calculate them is by a computer program. Anyway, we show the results of these calculations in table 3.

There we can see how the changes on the voting systems made some countries more powerful and others less powerful within the European Parliament, and why some of them wanted to change those systems and others did not.

We can compare the Banzhaf power indices with the share of the total population of each country. Some people hold the opinion that the power of a country should go only according to its population, and some others hold that there are more features to take into account, such as economical power or area. But, what do you think about it?

¹Data obtained from Eurostat (2003).

State	Population x 1000	Share of total
Germany	82536.7	18.158
France	59630.1	13.118
United Kingdom	59328.9	13.052
Italy	57321.0	12.610
Spain	41550.6	9.141
Poland	38218.5	8.408
The Netherlands	16192.6	3.562
Greece	11018.4	2.424
Portugal	10407.5	2.290
Belgium	10355.8	2.278
Czech Republic	10203.3	2.245
Hungary	10142.4	2.231
Sweden	8940.8	1.967
Austria	8067.3	1.775
Denmark	5383.5	1.184
Slovakia	5379.2	1.183
Finland	5206.3	1.145
Ireland	3963.6	0.872
Lithuania	3462.6	0.762
Latvia	2331.5	0.513
Slovenia	1995.0	0.439
Estonia	1356.0	0.298
Cyprus	715.1	0.157
Luxembourg	448.3	0.099
Malta	397.3	0.087
Total	454552.3	100

Table 2: Total population of each country and their shares of the EU-25 total population.

State	Population	Nice	13&60%	15&65%&B
Germany	18.158	8.5606	13.360	10.424
France	13.118	8.5600	9.4887	7.5805
United Kingdom	13.052	8.56	9.4281	7.5395
Italy	12.610	8.5600	9.1807	7.3818
Spain	9.141	8.1221	7.0202	5.8233
Poland	8.408	8.1221	6.7677	5.5566
The Netherlands	3.562	4.2284	3.6395	3.7619
Greece	2.424	3.9103	2.9610	3.3285
Portugal	2.290	3.9103	2.9040	3.2907
Belgium	2.278	3.9103	2.9040	3.2907
Czech Republic	2.245	3.9103	2.8470	3.2528
Hungary	2.231	3.9103	2.8470	3.2528
Sweden	1.967	3.2725	2.7328	3.1773
Austria	1.775	3.2725	2.6188	3.1015
Denmark	1.184	2.3102	2.2730	2.8766
Slovakia	1.183	2.3102	2.2730	2.8766
Finland	1.145	2.3102	2.2155	2.8389
Ireland	0.872	2.3102	2.1002	2.7632
Lithuania	0.762	2.3102	2.0423	2.7255
Latvia	0.513	1.3292	1.8102	2.5762
Slovenia	0.439	1.3292	1.8102	2.5762
Estonia	0.298	1.3292	1.7523	2.5384
Cyprus	0.157	1.3292	1.6943	2.5010
Luxembourg	0.099	1.3292	1.6360	2.4637
Malta	0.087	0.9933	1.6360	2.4637

Table 3: Banzhaf indices depending on the voting system.

Solutions to exercises

- Exercise 3.1.

1) $[0.5, 0.3, 0.1, 0.1]$, 2) $[0.4, 0.2, 0.2, 0.2]$.

The countries A and B prefer the second voting system (more than some 65% of the votes), country C prefers the first voting system (more than some 50% of the votes) and country D prefers the third voting system (at least three countries with more than some 65% of the votes), because those are the voting systems that give them more power according to the Banzhaf indices.

- Exercise 3.2. The Banzhaf power index of the standing members is 0.1669, and 0.0165 for the temporary members.

References

- [1] Banzhaf III, J. F. (1965) Weighted Voting Doesn't Work: A Mathematical Analysis, *Rutgers Law Review*, 19, 317-343.
- [2] Bilbao, J. M., Fernández, J. R., Jiménez, N., López, J.J. (2002) Voting power in the European Union enlargement, *European Journal of Operational Research*, 143, 181-196.